by the constant film model correction factor. Again it must be stressed for all other problems, e.g. uniform transpiration, that equation (13) has to be evaluated numerically.

CONCLUSIONS

In this note the film model has been applied to free convective heat transfer in the presence of mass transfer. The film model correction factor for heat transfer has been extensively compared with existing theoretical results of previous investigators. These elaborations concerned a laminar free convective boundary layer flow over an isothermal porous vertical plate. In these studies the governing equations were derived and solved numerically, the Prandtl number ranging from 0.7 to 1.

For uniform wall transpiration, the film model agreed with the literature within 5% for $-2.219 \leqslant \phi_t \leqslant 2.773$. The film model correction appeared to correlate with the same accuracy for a power-law distribution of the transpiration velocity and $-0.864 \leqslant \phi_t \leqslant 4.320$. For the said cases and ranges of ϕ_t the basic film model is well suited to describe the effect of mass transfer on free convective sensible heat transfer. Furthermore, these ranges of ϕ_t extend well beyond a large number of current practical applications.

In this note the classical film model for friction has not yet been applied to the examined physical process. For the studies referred to revealed that the exerted friction is reduced by both injection and large suction. The film model correction for friction, however, predicts enhanced and reduced friction for suction and injection, respectively (likewise heat transfer). This can be attributed to the momentum equation of the film model from which the correction factor is derived. In this equation the buoyancy and pressure gradient terms are omitted [1–4]. Though this neglect is allowed for forced convective flow, it is unacceptable for free convection. This insight may form a challenge to derive a friction correction factor from a film model with retained buoyancy and pressure gradient terms, and to apply this factor to free convective flow with transpiration.

Acknowledgements—The author wishes to thank the management of Akzo Research Laboratories Arnhem for their permission to publish this paper and Messrs. H. P. Korstanje and G. Vegt for their support of this work.

REFERENCES

1. A. C. Bannwart, Etude théoretique et expérimentale de la condensation d'une vapeur en présence d'incondensables, Thèse de Doctorat, Institut National Polytechnique de Grenoble (1988) [in French].
2. A. C. Bannwart and A. Bontemps, Condensation of a vapour with incondensables: an improved gas phase film model accounting for the effect of mass transfer on film thicknesses, *Int. J. Heat Mass Transfer* **33**, 1465–1474 (1990).
3. H. J. H. Brouwers and A. K. Chesters, Film models for transport phenomena with fog formation: the classical film model, *Int. J. Heat Mass Transfer* **35**, 1–11 (1992).
4. H. J. H. Brouwers, Film models for transport phenomena with fog formation, with application to plastic heat exchangers and condensers, Ph.D. Thesis, Eindhoven University of Technology (1990).
5. H. S. Mickley, R. C. Ross, A. L. Squyers and W. E. Stewart, Heat, mass, and momentum transfer for flow over a flat plate with blowing or suction, NACA Technical Note 3208 (1954).
6. C. Y. Wang and C. J. Tu, Effects of non-condensable gas on laminar film condensation in a vertical tube, *Int. J. Heat Mass Transfer* **31**, 2339–2345 (1988).
7. A. P. Colburn and T. B. Drew, The condensation of mixed vapours, *Trans. A.I.Ch.E.* **33**, 197–212 (1937).
8. D. R. Webb and R. G. Sardesai, Verification of multicomponent mass transfer models for condensation inside a vertical tube, *Int. J. Multiphase Flow* **7**, 507–520 (1981).
9. T. Mizushina, R. Ito, S. Yamashita and H. Kamimura, Film condensation of a pure superheated vapor inside a vertical tube, *Int. Chem. Engng* **18**, 672–679 (1978).
10. M. L. Corradini, Turbulent condensation on a cold wall in the presence of a noncondensable gas, *Nuclear Technol.* **64**, 186–195 (1984).
11. P. J. Vernier and P. Solignac, A test of some condensation models in the presence of a noncondensable gas against the Ecotra experiment, *Nuclear Technology* **77**, 82–91 (1987).
12. S. Ostrach, An analysis of laminar free convection flow and heat transfer about a flat plate parallel to the direction of the generating body force, NACA Technical Note 2635 (1952) or NACA Report 1111 (1953).
13. R. Eichhorn, The effects of mass transfer on free convection flow, *J. Heat Transfer* **82**, 260–263 (1960).
14. J. H. Merkin, Free convection with blowing and suction, *Int. J. Heat Mass Transfer* **15**, 989–999 (1972).
15. P. G. Parikh, R. J. Moffat, W. M. Kays and D. Bershader, Free convection over a vertical porous plate with transpiration, *Int. J. Heat Mass Transfer* **17**, 1465–1474 (1974).
16. E. M. Sparrow and R. D. Cess, Free convection with blowing or suction, *J. Heat Transfer* **83**, 387–389 (1961).
17. I. Mabuchi, The effect of blowing or suction on heat transfer by free convection from a vertical flat plate, *Bull. J.S.M.E.* **6**, 223–230 (1963).
18. J. F. Clarke, Transpiration and natural convection: the vertical plate problem, *J. Fluid Mech.* **57**(1), 45–61 (1973).

Int. J. Heat Mass Transfer. Vol. 35, No. 7, pp. 1844–1847, 1992
Printed in Great Britain
0017-9310/92 $5.00+0.00

Transient combined conduction and radiation in an absorbing emitting and anisotropically-scattering medium with variable thermal conductivity

CHUNG-JEN TSENG and HSIN-SEN CHU

Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan 30050, China

(*Received* 13 *January* 1991 *and in final form* 22 *May* 1991)

INTRODUCTION

THE ANALYSIS of steady [1–6] or unsteady [7–10] simultaneous radiation and conduction in an absorbing, emitting and scattering medium has received extensive attention in recent years due to its wide applications in engineering. While most previous works considered constant thermal conductivity, it is well known that the thermal conductivities of most nonmetal materials are not constant when subjected to moderate, say 100 K, or large temperature differences. Instead, the thermal conductivity of most nonmetal materials displays a linear relationship with temperature and can play a significant role on overall heat transfer. In a recent study, Chu and Tseng [11] investigated the effect of variable thermal conductivity on heat transfer of steady conduction–radiation interactions in an absorbing, emitting and scattering medium. Their results showed that the temperature dependence of thermal conductivity has significant effects on temperature distribution and heat transfer results.

The present work is concerned with the transient interaction of radiation and conduction in an absorbing, emitting and anisotropically-scattering slab with variable thermal conductivity. The major difficulty in analysis of such a problem stems from the nonlinear integral–differential characteristics of thermal radiative transfer. The present study utilizes the differential–discrete–ordinate (DDO) method [11, 12] to account for the radiation contribution. A fully implicit time-marching algorithm is employed to solve the time-dependent nonlinear energy equation. Emphases are given on the effect of temperature dependence of thermal conductivity on transient temperature, radiative and total heat flux distributions during interaction between conduction and radiation in the medium.

FORMULATION

Consider an absorbing, emitting and anisotropically-scattering planar medium that is initially at a uniform temperature T_0. For times greater than zero, the boundary surfaces are kept at specified temperatures T_1 and T_2, respectively. Both diffuse and specular reflections at boundaries are included. Under these conditions, the dimensionless transient energy equation can be expressed as

$$\frac{\partial\theta}{\partial\xi} = (1+\gamma\theta)\frac{\partial^2\theta}{\partial\tau^2}+\gamma\left[\frac{\partial\theta}{\partial\tau}\right]^2-\frac{1-\omega}{N}\left[\theta^4-\frac{1}{2}\int_{-1}^{1}\psi\,d\mu\right], \quad (1)$$

associated with the following initial and boundary conditions

$$\theta(0,\xi)=1,\quad \theta(\tau_0,\xi)=\theta_2,\quad \theta(\tau,0)=\theta_0 \quad (2)$$

where we have introduced the following dimensionless quantities:

$$N=\frac{\sigma_e k_0}{4n^2\sigma T_1^3},\quad \gamma=\frac{\gamma' T_1}{k_0},\quad \xi=\frac{k_0\sigma_e^2 t}{\rho c_p},$$

$$Q^r(\tau,\xi)=\frac{q^r}{n^2\sigma T_1^4/\pi},\quad \psi(\tau,\mu,\xi)=\frac{I}{n^2\sigma T_1^4/\pi} \quad (3)$$

and $\tau=\sigma_e y$ is the optical path, ω the single scattering albedo, and $\theta=T/T_1$. In equations (3), the linear relationship, $k=k_0+\gamma' T$, is used to investigate the effects of variable conductivity. The dimensionless intensity ψ satisfies the radiative transfer equation

$$\mu\frac{d\psi(\tau,\mu)}{d\tau}=(1-\omega)\theta^4(\tau)-\psi(\tau,\mu)+\frac{\omega}{2}\int_{-1}^{1}\psi(\tau,\mu')p(\mu',\mu)\,d\mu', \quad (4)$$

with the boundary conditions

$$\psi(0,\mu)=\varepsilon_1+\rho_1^s\psi(0,-\mu)+2\rho_1^d\int_0^1\psi(0,-\mu')\mu'\,d\mu',\quad \mu>0, \quad (5a)$$

$$\psi(\tau_0,-\mu)=\varepsilon_2\theta_2^4+\rho_2^s\psi(\tau_0,\mu)+2\rho_2^d\int_0^1\psi(\tau_0,\mu')\mu'\,d\mu',\quad \mu>0, \quad (5b)$$

where ε is the emissivity, and ρ^s and ρ^d are the specular and diffuse reflectivities, respectively.

METHOD OF SOLUTION

Equations (1) and (4) are coupled through θ and ψ and have to be solved simultaneously. It is very difficult to obtain analytic solutions of this very complicated nonlinear system. In this work, the numerical DDO method is used to solve the radiative transfer portion of the problem and a fully implicit time-marching algorithm is employed to deal with the nonlinear energy equation.

After replacing the integral over μ' in equations (4) and (5) by quadratures, equation (4) is reduced to the following system of ordinary differential equations [11, 12]:

$$\mu_i\frac{\partial\psi_i(\tau,\xi)}{\partial\tau}=(1-\omega)\theta^4-\psi_i(\tau,\xi)+\frac{\omega}{2}\sum_{j=-M}^{M}w_j\psi_j(\tau,\xi)\Phi_{ji},\quad i=-M,\ldots,M \quad (6)$$

where μ_i are quadrature points, w_i the corresponding weights, $\psi_i(\tau,\xi)=\psi(\tau,\mu_i,\xi)$, and $\Phi_{ji}=\Phi(\mu_j\to\mu_i)$. In this investigation, linear anisotropic scattering is considered to examine the effect of scattering anisotropy. The linearly anisotropic scattering phase function is expressed as $\Phi_{ji}=1+A_1\mu_j\mu_i$, where $A_1\to 1$ represents strong forward scattering and $A_1\to -1$ corresponds to strong backward scattering. Similarly, the energy equation, equation (1), is reduced to

$$\frac{\partial\theta}{\partial\xi}=(1+\gamma\theta)\frac{\partial^2\theta}{\partial\tau^2}+\gamma\left[\frac{\partial\theta}{\partial\tau}\right]^2-\frac{1-\omega}{N}\left[\theta^4-\frac{1}{2}\sum_{j=-M}^{M}w_j\psi_j\right]. \quad (7)$$

After differencing the unsteady term $\partial\theta/\partial\xi$ using three-term backward difference, equation (7) becomes

$$(1+\gamma\theta)\frac{d^2\theta}{d\tau^2}=\frac{1}{\Delta\xi}(1.5\theta^l-2\theta^{l-1}+0.5\theta^{l-2})-\gamma\left[\frac{d\theta}{d\tau}\right]^2+\frac{1-\omega}{N}\left[\theta^4-\frac{1}{2}\sum_{j=-M}^{M}w_j\psi_j\right], \quad (8)$$

where $\Delta\xi$ is the non-dimensional time step, and l, $l-1$, and $l-2$ are the present, the last, and the one before last times, respectively. The partical differential equation is transformed into an ordinary differential equation (ODE) for a given time.

Equations (6) and (8) constitute a system of $2M$ first order ODEs in ψ_i and a second order nonlinear ODE in θ. The $2M$ boundary conditions for ψ_i are supplied by

$$\psi_i(0,\xi)=\varepsilon_1+\rho_1^s\psi_{-i}(0,\xi)+2\rho_1^d\sum_{j=1}^{M}w_j\psi_{-j}(0,\xi)\mu_j,\quad i=1,\ldots,M \quad (9a)$$

$$\psi_{-i}(\tau_0,\xi)=\varepsilon_2\theta_2^4+\rho_2^s\psi_i(\tau_0,\xi)+2\rho_2^d\sum_{j=1}^{M}w_j\psi_j(\tau_0,\xi)\mu_j,\quad i=1,\ldots,M \quad (9b)$$

and the boundary conditions for θ are specified by equation (2).

After separating the second order ODE into two first order ODEs, the resulting system of $2M+2$ first order ODEs is solved by using subroutine BVPFD from commercial software IMSL. The subroutine BVPFD solves a two-point boundary value problem using a variable order finite difference method with deferred corrections. Once the temperature and radiative intensity distributions are known, the radiative heat flux is obtained from

$$Q^r(\tau,\xi)=2\pi\sum_{j=-M}^{M}w_j\psi_j(\tau,\xi)\mu_j, \quad (10)$$

and the total heat flux in the medium is given by

$$Q^t(\tau,\xi)=\frac{q^t(\tau,\xi)}{k_0\sigma_e T_1}=-(1+\gamma\theta)\frac{\partial\theta(\tau,\xi)}{\partial\tau}+\frac{Q^r(\tau,\xi)}{4\pi N}. \quad (11)$$

The marching procedure is continued until the steady state is assured. In the present study, the steady state is defined as both

Table 1. Comparison of the transient temperature distribution and radiative heat flux with isotropic scattering at $\xi = 0.05$, $\tau_0 = 1$, $\theta_1 = 1$, $\theta_2 = \theta_0 = 0$, $N = 0.1$, $\gamma = 0$ and $\omega = 0.5$ for several wall reflectivity cases

ε_1	ρ_1^d	ε_2	ρ_2^d	Investigators	θ at $\tau = 0.25$	θ at $\tau = 0.50$	$Q^r/4\pi N$ at $\tau = 0.25$	$Q^r/4\pi N$ at $\tau = 0.50$
1	0	0	1	Sutton [8]	0.4888	0.1778	1.9304	1.3305
1	0	0	1	Tsai and Lin [10]	0.4889	0.1773	1.9328	1.3292
1	0	0	1	Present	0.4896	0.1773	1.9350	1.3297
1	0	1	0	Sutton [8]	0.5030	0.2005	1.6279	0.8639
1	0	1	0	Tsai and Lin [10]	0.5031	0.2001	1.6308	0.8629
1	0	1	0	Present	0.5038	0.2001	1.6317	0.8624
0.5	0.5	0.5	0.5	Sutton [8]	0.4671	0.1591	0.9944	0.7018
0.5	0.5	0.5	0.5	Tsai and Lin [10]	0.4671	0.1585	0.9957	0.7002
0.5	0.5	0.5	0.5	Present	0.4676	0.1584	0.9989	0.7000

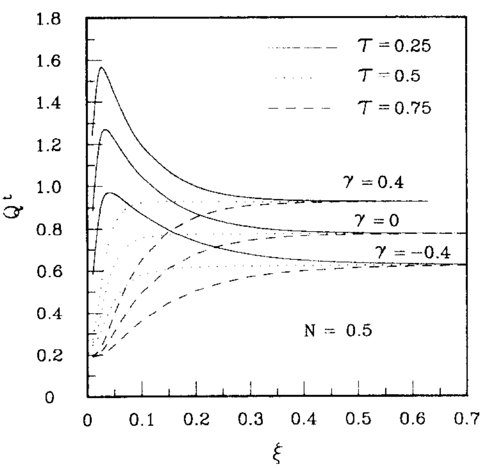

FIG. 1. Effects of γ on the histories of total heat flux at three positions and $\varepsilon_1 = \varepsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = \theta_0 = 0.5$ and $\omega = 0.5$.

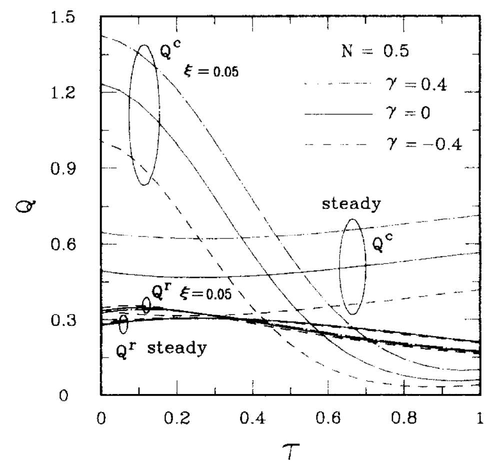

FIG. 2. Effects of γ on the conductive and radiative heat flux distributions at two times and $\varepsilon_1 = \varepsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = \theta_0 = 0.5$, $\omega = 0.5$, $A_1 = 1$.

$$|\theta_i^j - \theta_i^{j-1}| < 1\times 10^{-4} \quad \text{and} \quad |Q_i^{t,j} - Q_i^{t,j-1}| < 5\times 10^{-4} \qquad (12)$$

being met at all mesh points. The results were obtained by carrying out the numerical algorithm on a VAX 8800 at National Chiao Tung University with 16 Gaussian quadrature points and 41 uniformly spaced grids.

RESULTS AND DISCUSSION

Although the present fully implicit time-marching finite difference technique is unconditionally stable, there are time step constraints to yield physically realistic solutions. A numerical experiment is carried out here to ensure the independence of the numerical results on the time step $\Delta\xi$. The time step is determined to be 0.0016667 for all cases.

To illustrate the accuracy of the numerical solutions of the present work, results of non-dimensional temperatures and heat transfer are compared with the hybrid Galerkin [8] and nodal approximation [10] methods for isotropic scattering and constant thermal conductivity. Table 1 shows the comparisons at $\tau_0 = 1$, $\theta_2 = \theta_0 = 0$, $N = 0.1$, $\gamma = 0$ and $\xi = 0.05$. It is shown that the transient temperature and net radiative flux results of the present method are in excellent agreement with all the results listed. In contrast to the other methods, the DDO method is easier to program and any degree of scattering anisotropy and temporal and spatial variations of radiative properties can be easily incorporated.

Table 2. Effect of temperature dependence of thermal conductivity on the dimensionless time required to reach steady state (ξ_S) at $\varepsilon_1 = \varepsilon_2 = 1$, $\omega = 0.5$, $\tau_0 = 1$, $\theta_2 = \theta_0 = 0.5$, $A_1 = 1$ and $N = 0.5$

γ	ξ_S
0	0.7783
0.4	0.6283
−0.4	1.0233

The effects of variable thermal conductivity on the histories of total heat flux at three positions and on the time required to reach steady state (ξ_S) for isotropic scattering at $\varepsilon_1 = \varepsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = \theta_0 = 0.5$ and $\omega = 0.5$ are illustrated in Fig. 1 and Table 2, respectively. It is shown that more time is required for a negative γ system to reach steady state than a positive one. This is because a positive γ medium transports more conductive and more total heat, as shown in Figs. 1 and 2, from the hot region to the cold region than a negative γ medium and thus leads to a faster balance in energy. Figures 2 and 3 demonstrate the effects of temperature dependence of thermal conductivity on temperature and heat flux distributions at $\xi = 0.05$ and steady state. The effect of γ on conductive heat flux distribution is more significant than that on radiative heat flux distribution while the effect of γ on the temperature distribution is more expressive at $\xi = 0.05$ than at steady state.

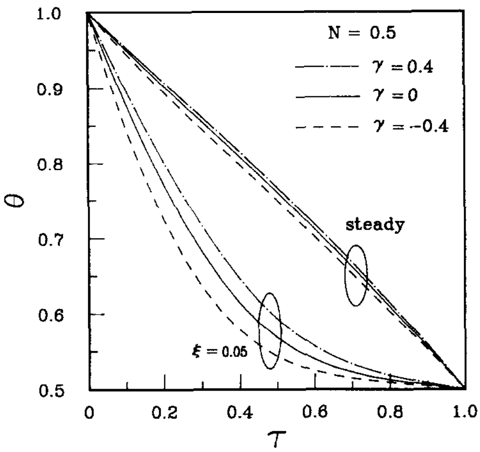

FIG. 3. Effects of γ on the temperature distribution at two times and $\varepsilon_1 = \varepsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = \theta_0 = 0.5$, $\omega = 0.5$, $A_1 = 1$.

CONCLUDING REMARKS

A study has been made of the transient transfer of energy due to the combined effects of conduction and radiation for an absorbing, emitting and anisotropically-scattering medium with temperature-dependent thermal conductivity. The DDO method is utilized to solved the radiative transfer equation while a time-marching algorithm is proposed to deal with the unsteady energy equation. It is shown in the present numerical analysis that the temperature dependence of thermal conductivity of the medium has significant effects on both temperature and heat flux distributions of the problem considered.

Acknowledgement—This work was supported by the National Science Council of R.O.C. through grants NSC 79-0401-E009-14.

REFERENCES

1. R. Viskanta and R. J. Grosh, Heat transfer by simultaneous conduction and radiation in an absorbing medium, *J. Heat Transfer* **84**, 63–72 (1962).
2. J. B. Bergquam and R. A. Seban, Heat transfer by conduction and radiation in absorbing and scattering materials, *J. Heat Transfer* **93**, 236–239 (1971).
3. A. L. Crosbie and R. Viskanta, Interaction of heat transfer by conduction and radiation in a nongray planar medium, *Wärme- und Stoffübertragung* **4**, 205–212 (1971).
4. A. Dayan and C. L. Tien, Heat transfer in a gray planar medium with linear anistropic scattering, *J. Heat Transfer* **97**, 391–396 (1975).
5. W. W. Yuen and L. W. Wong, Heat transfer by conduction and radiation in a one-dimensional absorbing, emitting and anisotropic-scattering medium, *J. Heat Transfer* **102**, 303–307 (1980).
6. H. S. Chu and C. J. Tseng, Thermal performance of ultra-fine powder insulations at high temperatures, *J. Thermal Insulation* **12**, 298–312 (1989).
7. T. W. Tong, D. L. McElroy and D. W. Yarbrough, Transient conduction and radiation heat transfer in porous thermal insulations, *J. Thermal Insulation* **9**, 13–29 (1985).
8. W. H. Sutton, A short time solution for coupled conduction and radiation in a participating slab geometry, *J. Heat Transfer* **108**, 465–466 (1986).
9. J. W. Rish III and J. A. Roux, Heat transfer analysis of fiberglass insulation with and without foil radiant barriers, *J. Thermophys. Heat Transfer* **1**, 43–49 (1987).
10. J. H. Tsai and J. D. Lin, Transient combined conduction and radiation with anisotropic scattering, *J. Thermophys. Heat Transfer* **4**, 92–97 (1990).
11. H. S. Chu and C. J. Tseng, Combined conduction and radiation in an absorbing, emitting and anisotropically scattering medium with temperature-dependent thermal conductivity, to appear in *J. Thermophys. Heat Transfer* (1991).
12. S. Kumar, A. Majumdar and C. L. Tien, The differential–discrete–ordinate method for solutions of the equation of radiative transfer, *J. Heat Transfer* **112**, 424–429 (1990).

Int. J. Heat Mass Transfer. Vol. 35, No. 7, pp. 1847–1855, 1992
Printed in Great Britain
0017-9310/92 $5.00+0.00
Pergamon Press Ltd

Transfert de chaleur et limite de mouillabilite d'un film ruisselant sur paroi plane inclinée entre 0° et 90°

BAI-QI LIU et PIERRE LE GOFF
Laboratoire des Sciences du Génie Chimique, CNRS-ENSIC-INPL, Nancy, France

(*Reçu le* 7 *Février* 1991 *et sous forme finale le* 12 *Juin* 1991)

1. INTRODUCTION

LA TECHNIQUE du film ruisselant pour le traitement thermique des fluides est largement utilisée dans les industries chimiques et para-chimiques. Cette technique présente cependant plusieurs difficultés pratiques de réalisation :

(1) Des *zones sèches* se forment sur la paroi, dés que le débit de liquide est trop petit ou que le flux de chaleur est trop grand.

(2) Le *distributeur*, destiné à répartir le liquide sous forme d'un film mince, d'épaisseur uniforme, est d'un fonctionnement délicat.

De nombreux auteurs ont cherché à éliminer ces défauts et aussi à augmenter les coefficients de transfert de chaleur et de matière, en ajoutant des promoteurs de turbulence sur la paroi, ou bien en traitant cette paroi même pour la rendre rugueuse.

Mais ces travaux ont été limités à l'étude de film ruisselant sur une paroi *verticale* d'un tube *cylindrique*. Or, il n'est pas facile de transposer ces résultats au cas de film ruisselant sur une paroi *plane*, *inclinée* d'un angle varié entre 0° et 90°, par rapport à l'horizontale. D'ailleurs, les procédés comme la distillation de l'eau de mer en système multi-effets, la distillation de solutions diluées et polluées industrielles en multi-